// MONTHLY WEATHER REVIEW

Editor, ALFRED J. HENRY

VOL. 58, No. 5
W. B. No. 1016

MAY, 1930

CLOSED JULY 3, 1930
ISSUED AUGUST 8, 1930

PREDICTING MINIMUM TEMPERATURE, ESPECIALLY AS A FUNCTION OF PRECEDING TEMPERATURE

By ESEK S. NICHOLS

[Weather Bureau office, San Jose, Calif., March 12, 1930]

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INTRODUCTION

The fact that the morning minimum temperature is a function of several variables, the effects of some of them obscure, is shown not only by theoretical considerations but also by the variety of methods that have been proposed for its prediction, as well as by the considerable scattering of dots above and below the lines of best fit on all minimum-temperature dot charts, even those prepared for restricted conditions.

PREDICTION FROM THE EVENING DEW POINT

Some early studies of minimum-temperature prediction used preceding dew point as a standard of reference; e. g., in 1910 O'Gara reported that in the Rogue River Valley of Oregon, under certain conditions "there is a relation existing between the dew-point temperature observed in the early evening and the minimum temperature of the following morning." (1). He stated equality of dew point and ensuing minimum under certain very restricted conditions; and mentioned other conditions that are followed by minima above or below the dew point, although he did not evaluate the effects of these latter conditions to any extent. The restrictions and modifications placed by O'Gara upon the use of the relation stated by him were necessitated by the fact that the dew point alone does not determine the ensuing minimum temperature. (See also (2), (2A), (3), and (4).)

Other contributions to the subject of direct dew point minimum-temperature relations are referred to by Ellison, who concludes that the dew-point formula is "inherently faulty." His conclusion depends on the inference that the formula in question is based on the easily-disproved principle that "the minimum temperature would not be lower than the temperature of the evening dew point" (5). However, this "principle" differs widely from O'Gara's expression of a "relation" and from my consideration of the minimum temperature as a "possible function" of the dew point. (**3**) (p. 213). In fact, the hygrometric minimum-temperature formulas simply state that at a given relative humidity the ensuing minimum temperature is a function of the dew point; and thus these formulas are essentially modifications and extensions of the general dew point formula (**12**).

HYGROMETRIC FORMULAS AND GRAPHS

As has been stated many times, it is customary to derive a hygrometric formula or curve by finding the line of best fit to the employed data as plotted on cross-section paper, relative humidities as abscissas and departures of ensuing minimum temperature from preceding dew point as ordinates. In early investigations straight-line relations were stated; but this method was soon found to be inaccurate, and parabolic curves and equations were fitted to the dot charts (**12**).

Still later I found that the rectangular [1] hyperbola having asymptotes parallel to the axes of coordinates is in many cases preferable to the parabola, and subsequent experience has convinced me that it is generally preferable. However, if the original dot chart on which the curve of best fit has been drawn is used in actual forecasting, as suggested previously, it is unimportant whether the curve be of parabolic, hyperbolic, or other form, since results will be taken directly from the graph without computation. Therefore, a free-hand curve, its course guided by computations as described below, will often be most desirable (**9**).

METHOD OF ARBITRARY CORRECTIONS

Young employs as his basic formulas straight-line equations of the Donnel type, to which are applied series of corrections for groups of values of dew point and relative humidity separately. Ellison, in the paper already referred to, develops from data for Medford, Oreg., a set of straight-line formulas after the Donnel-Young method. In order to make these formulas fit the data more closely he applies a series of arbitrary corrections, according to Young. The formula (**8**) found for clear weather is,

$$y = d - \frac{h-20}{4} + V_a + V_h$$

d and h are evening dew point and relative humidity, respectively, y is the indicated minimum temperature, and V_d and V_h are corrections for groups of dew point and relative humidity values, as follows: [2]

d	V_a	h	V_h
°		*Per cent*	
7–24	+10	12–21	0
25–29	+3½	22–26	−½
30–35	+2	27–30	−1
36–43	−2	31–39	+1
---------	---------	40–51	−1

When E_d and E_h are each equal to O we have the base line of the formula, a straight line whose equation is

$$Y = y - d = -\frac{h-20}{4} = 5 - \frac{h}{4}.$$

[1] Not "rectilinear" as erroneously stated by Ellison in (**5**), p 490. The curve is the rectangular or equilateral hyperbola having asymptotes perpendicular to each other. [The word "rectangular" was misprinted "rectilinear".—Ed.]

[2] Temperatures in degrees Fahrenheit are used in this formula and others considered in the present paper.

This the lower straight line of Ellison's Figure 2, reproduced as the line AB of Figure 1, herewith. We may now represent the corrections E_h graphically by modifying the course of AB to form the irregular line Abc . . . . . B. We may then raise or lower this irregular line bodily through the number of spaces corresponding to each dew-point correction, V_d, and obtain the irregular lines A_1 B_1, A_2 B_2, A_3 B_3, A_4 B_4, which completely express Formula 8 for clear weather graphically. Similar sets of irregular lines would be obtained by plotting Ellison's formulas and corrections for partly cloudy and for cloudy weather.

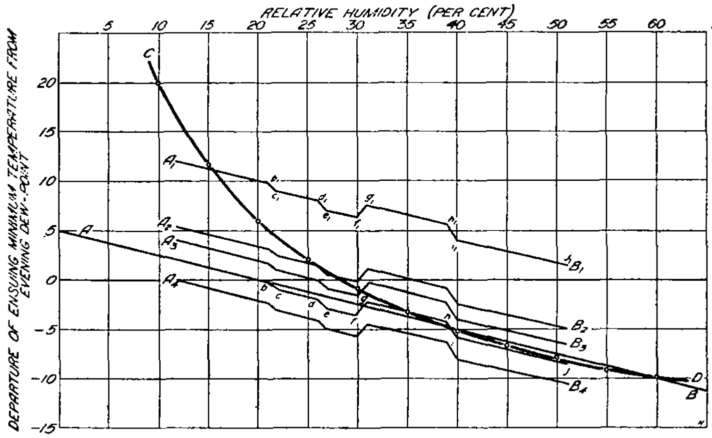


FIGURE 1.—Donnel-Young and hyperbolic curves from Medford data

Using Ellison's data for Medford, Oreg., plotted on his Figure 1, let us now determine a hyperbolic formula for that station. Pass a rectangular hyperbola through the points (10, 20), (30, − 1), and (60, − 10), which are taken as "star points" according to the method previously described (*9*). We obtain the equation $(X+10)(Y+22) = 840$, or

$$Y = \frac{840}{X+10} - 22$$

which is the equation of a rectangular hyperbola having as asymptotes the lines $X = -10$ and $Y = -22$. The curve is drawn as the line CD on Figure 1.

Figure 1 serves at least two purposes. First, especially if we transfer lines AB and CD to Ellison's Figure 1, we may compare the parabola with the hyperbola and with the arbitrary-correction formula. Second, the reader may by actual trial determine for himself whether it is more convenient to obtain the indicated minimum temerature by substitution of proper values in a formula or, as I have suggested, to read the minimum-temperature departure directly from the graph of the formula as drawn on Figure 1.

We are unable to agree with Ellison in the claim that the arbitrary-correction method as described by him is more accurate than others. The parabola, hyperbola, or other form of curve should be considered as a basic or general curve. If at any station we find a consistent variation from the basic curve when used under certain values of dew point, for instance, we have only to draw other curves above or below the standard curve or to prepare another chart for use with such special dew-point values, as was exemplified for certain stations in Supplement 16 of MONTHLY WEATHER REVIEW.

Thus the curve CD occupies the same position in the hyperbolic system of curves for Medford as does the irregular line Abc . . . . . B in the arbitrary-correction system. A comparison of these two lines with the data of Ellison's Figure 1 is not particularly creditable to the latter, which differs comparatively little from the Donnel straight line AB of our Figure 1. However, as a result of previous experience with data for other stations, it may be stated with confidence that the supplementary hyperbolic curves that would be obtained if dew points were available for each separate Medford observation used by Ellison, would differ much less from their basic curve than do the final arbitrary-correction curves from their basic line Abc . . . . B, thus the advantage found at first for the hyperbolic form, would be lost. Our special curves would or would not be parallel to each other and to the basic curve according to indications of the plotted data. If the data showed that supplementary curves would be useful, we should draw and use them; otherwise we should use the basic curve. We conclude, then, that the hyperbolic may be made equally as accurate as the arbitrary-correction method when properly applied.

Using the same Medford data as before, Ellison derives a "Nichols free-hand curve formula" and applies the method of arbitrary corrections thereto. For values of h he finds corresponding corrections V_h, but these "corrections" are evidently averages of ordinates of all plotted points having the abscissa h specified in each case. Thus he refers his curve to the X-axis instead of to the basic line AB, as in Young's method, and simplifies the formula by thus eliminating the expression $\frac{h-20}{4}$, or similar fraction. The corrections, V_d, given for groups of values of dew point, raise or lower the basic curve (of the Nichols free-hand formula) bodily, the same as in case of Ellisons formula 8; and thus produce a family of special curves parallel to each other and to the basic curve, which method is evidently inferior to that of drawing supplementary curves, already described, parallel to basic curve or not, according to data.

When h has the values 20, 21, 22, and 23, he obtains the following values for V_h; 4½, 3½, 4, and 3, respectively. When these data are plotted on cross-section paper, an S-shaped irregularity appears in the curve through the dots. He states that "the line of best fit is irregular," and that "the addition of the method of arbitrary corrections will produce the same irregular line of best fit to any hygrometric dot-chart" (see (**5**), p. 493), evidently meaning that the line of best fit is any line, regardless of form, that passes through or nearest to the greatest number of dots on the chart. However, the S-shaped irregularity referred to does not appear in plotted forms of Ellison's formulas 7 and 8; and the irregularities in the latter do not appear in the corrected Nichols free-hand curve, although derived from the same data, using the method of arbitrary corrections.

The fact that the basic line used by Ellison in obtaining V_h for the free-hand curve is the X axis, while he uses other straight lines in formulas 7 and 8, in itself makes no difference in the final curve obtained, as he very well indicates. Rather, he fails to obtain the same line of best fit because of different methods of handling the data. Thus, in the free-hand case V_h is determined for each per cent of relative humidity from 12 to 52; in other cases for groups of relative-humidity values. Also, in the former case, only one principal curve is derived; in the latter, three, one each for clear, partly cloudy, and cloudy weather. Therefore, his statistical comparison of the two systems appears improper and unconvincing.

NECESSITY OF SMOOTHING HYGROMETRIC CURVES

We may concede that irregular lines, such as he employs, may fit the data plotted on a particular dot chart, such as that for Medford, better than a smoothed line of continuously decreasing negative slope without sudden bends or angles, while at the same time we maintain the position that the smoothed line accords better with physical law. The fact that "upward bends and twists" occur at times in thermograms, as Ellison states in memorandum relative to my position, is immaterial; since the hygrometric dot chart does not show the course of temperature fall through the night, but the morning minimum temperature that follows certain evening dew point and relative humidity values.

Consider, for example, the S-shaped irregularity already referred to. Assume three different evenings on which the dew point had the same value, 30. On the first evening let the relative humidity be 22 per cent (the evening current temperature being, therefore, 70); the indicated morning minimum is, then, $30+4=34$ according to the S-shaped curve. On the second evening let the relative humidity be 23 per cent (current temperature, therefore, 69); the indicated minimum is $30+3=33$. In the third case let the relative humidity be 21 per cent (current temperature, 71); indicated minimum, $30+3\frac{1}{2}=33\frac{1}{2}$. That is, starting with the first case, we obtain an indicated decrease of ensuing minimum temperature whether we increase or decrease the relative humidity (i. e., decrease or increase, respectively, the current evening temperature). Similar anomalies are obtained by using other dew points than 30 with the same relative humidities as above. (We have used the principal—curve S—irregularity for convenience, without affecting the character of results.) Therefore, this S irregularity, and others of similar nature, require explanation, not being in accord with the general hygrometric minimum-temperature law that increase of evening relative humidity is followed by a decrease of morning minimum temperature, evening dew point being the same in both cases.

It is apparent that the S-shape is accidental—perhaps the result of one or two abnormal occasions—and would be smoothed out with increase of available data. If the values, V_h, were plotted and then smoothed, after the method used in my paper on wind velocities at New York City (**11**), the use of arbitrary corrections would appear as an improved method of locating the free-hand curve of best fit. In fact, by this method we can, in general, produce a better line than any hyperbola, parabola, or other line that can be expressed by a simple equation.

PREFERRED METHOD

We conclude, then, that, in general, the most accurate and convenient method of employing the hygrometric relation in minimum-temperature forecasting is as follows: Draw a smooth free-hand curve directly on the dot-chart, deciding the course of the line, not simply by the eye, but also by considering average values at each ordinate. Then supplementary curves such as data indicate would be useful may be drawn, parallel to the principal line or not according to data, for use under particular values of the evening dew point (and of other meteorological conditions as well). Averages, used in locating both principal and special curves, may be simply arithmetical means, but would better be medians or weighted means; least-square methods might be used in finding the best values in certain cases. If simple equations, such as those of parabolas or hyperbolas, can be fitted to our curves, the fact is interesting and perhaps of value; but knowledge of the equations will evidently increase neither accuracy nor convenience of prediction.

RELATION BETWEEN DEW POINT, RELATIVE HUMIDITY, AND CURRENT TEMPERATURE

Let us examine the relation, already referred to, between relative humidity and current temperature, dew point keeping constant values. From the definition of relative humidity we have

$$e=\frac{h}{100(E)}$$

where h is relative humidity, E is the vapor pressure of saturation at the current temperature, t, and e is the cur-

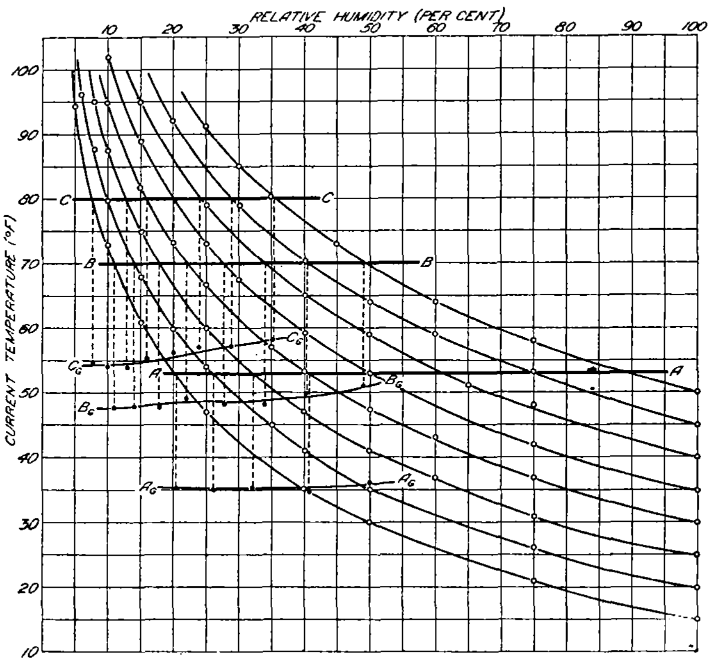


FIGURE 2.—Relation between dew point, relative humidity, and current temperature

rent vapor pressure. e is a function of the dew point, d, and may be obtained from vapor-pressure tables when d is given (**10**). If h be given also, we can now find E by substituting for e and h in the above equation. Having obtained E, we may now interpolate the value of t from vapor-pressure tables; e. g., having given the dew point, 30°, and relative humidity, 40 per cent, find the current temperature, t. From (**10**) we find

$$E=\frac{100\times 0.164}{40}=0.410$$

which corresponds to a temperature of approximately 53.5°. Keeping the dew point constant at 30°, we find that at 20 per cent relative humidity t is 73; while with h at 75 per cent, t is 37, etc. Values of t obtained thus for dew points at 5° intervals from 15 to 50 have been plotted on Figure 2, above; then smooth curves were drawn for each dew-point value, eight curves in all. Having d and h, we may now interpolate t directly from these curves.

TRANSFORMATION OF THE HYGROMETRIC FORMULAS TO THE CURRENT - TEMPERATURE FORM — ALGEBRAIC METHOD

The hygrometric minimum-temperature relation we have discussed may be expressed in general terms, thus:

$$y-d=F(h) \qquad \text{(Form A)}$$

where y, d, and h represent the same elements as above, whatever be the form of the curve of best fit. If d be variable we have

$$y=f(h, d) \qquad \text{(Form B)}$$

From the definition of relative humidity we have, as before,

$$h=100\,\frac{e}{E}$$

If the dew point be constant, e is also constant, and we have $h=G(t)$, since E is a function of the current temperature, t. Then Form A reduces to

$$y=\phi(h)=\psi(t) \qquad \text{(Form C)}$$

That is, in such cases, the minimum temperature is a function of the current temperature at the time of the (evening) observation. We may then transform any hygrometric equation from the usual Form A to the temperature Form C by substituting in the former the value of h obtained from the relation, $h=G(t)$, which may be expressed for a particular dew point as the equation of the curve for that dew point plotted on our Figure 2. Thus, fitting a rectangular hyperbola to the curve for dew point 30°, we have the following equation:

$$(h+24)(t-5.5)=3{,}037.5$$

or

$$h=\frac{3{,}037.5}{t-5.5}-24.$$

Substituting this value of h in the hyperbolic equation for Grand Junction, Colo. (p. 499 of (9)), we have

$$Y=\frac{738}{h+8}-7=\frac{738}{\frac{3{,}037.5}{t-5.5}-24+8}-7=$$

$$\frac{738\,t-4{,}059}{3{,}125.5-16\,t}-7=\frac{140{,}105}{3{,}125.5-16\,t}-53=$$

$$-\frac{8{,}757}{t-195}-53$$

approximately, from which

$$(Y+53)(t-195)=-8{,}757$$

which is the equation of another rectangular hyperbola. Differentiating, we have

$$\frac{dY}{dt}=\frac{8{,}757}{(t-195)^2}$$

slope of the curve; and

$$\frac{d^2Y}{dt^2}=\frac{-2\ (,757)\ (t-195)}{(t-195)^4}=\frac{-17{,}514}{(t-195)^3}$$

rate of change of slope, which is positive when t lies between 0 and 195. When t is small, $\frac{d^{2Y}}{2}$ is also small; even when t is as great as 95, $\frac{d^{2d^{t}}Y}{dt^2}$ is less than 1/57. Therefore, when the current temperature, t, lies between 0° and 95° the slope increases very slowly and we have, practically, a straight line.

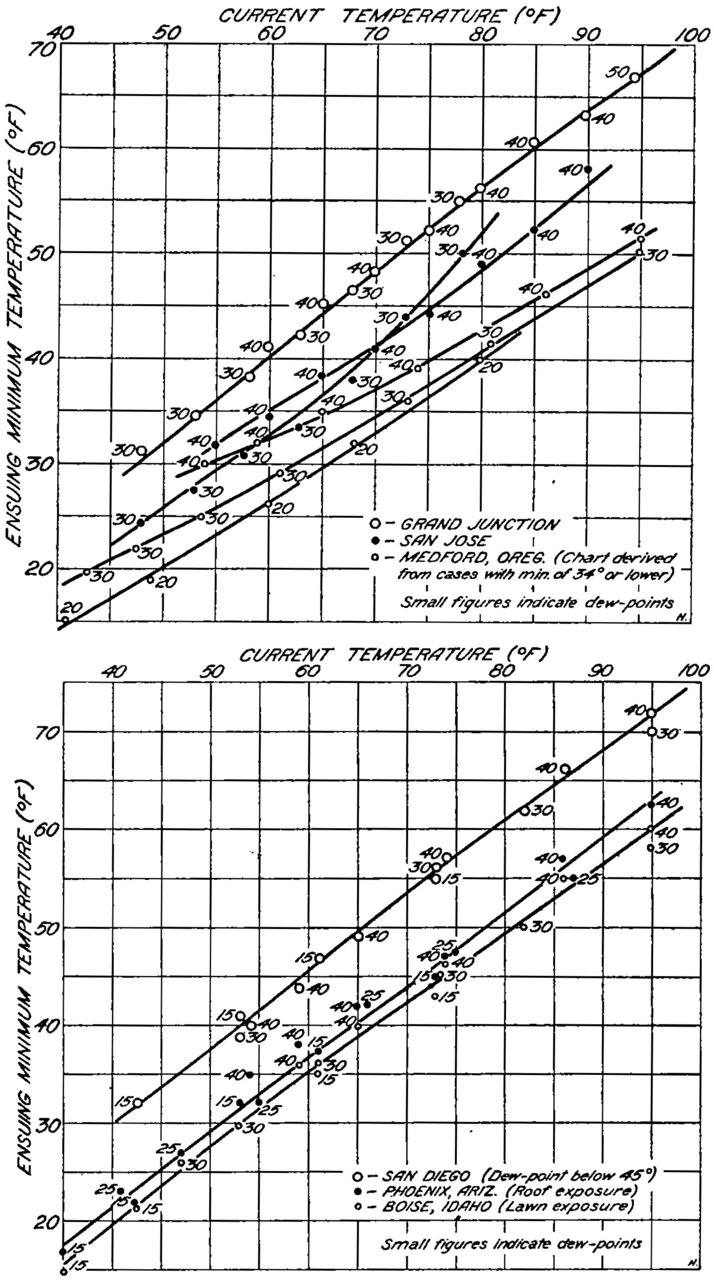
GRAPHICAL TRANSFORMATION

We can more readily transform the hygrometric equation of the Form A to the temperature form C by graphical methods. Assuming any particular dew point and relative humidity, we determine from the hygrometric formula or curve for the particular station and conditions in mind, the corresponding indicated minimum temperature, and from our curves of Figure 2 the corresponding current temperature. Then plot on a dot chart the current-temperature and minimum-temperature values so found. If we keep the dew point constant (i. e., consider other cases with the same evening dew point) and allow the relative humidity, and consequently the current (evening) temperature, to vary, we obtain a series of corresponding current-temperature and minimum-temperature values which may be plotted as a series of points on our dot chart. We may then draw through these points a line for use when the dew point is of the value chosen. A series of points so obtained for dew point 30° and another series for 40° are plotted on Figure 3, using the same noon hygrometric chart for Grand Junction, Colo., as was used in obtaining the hyperbolic equation already employed for that station; and we find that the points we have plotted lie on a straight line, practically. This agrees with the result already obtained by algebraic substitution. Not only this; but it is noted, also, that the points for 40° dew point lie on practically the same line as those for 30°. Hence, one straight line has been drawn for use with both dew points.

On Figure 3 have been plotted, also, points obtained similarly for the same dew points, using the curve for San Jose, Calif. (9, p. 500). The two sets of points lie on lines that are nearly straight, though some tendency to hyperbolic form is noted; but the two lines intersect, showing that increase of dew point does not, at all current evening temperatures, cause an increase in the minimum indicated by this San Jose chart. Further, on Figure 3, have been plotted similar data for Medford, Oreg., using the hyperbolic curve of Figure 1, for dew points 20°, 30°, and 40°. Each set of points lies on a practically straight line; the three lines are nearly coincident at higher temperatures, but diverge somewhat at lower. On Figure 4 we have similar sets of points for 15°, 25°, or 30°, and 40° dew point at Phoenix, Ariz. (roof exposure); Boise, Idaho (lawn exposure); San Diego, Calif. (dew point below 45), using charts for those stations in Supplement 16; the points for each station lie on or near a practically straight line drawn on the chart.

In all cases on Figures 3 and 4 the hygrometric charts used have been presented by their authors for use at any dew point, at least none was specified, except in the case of San Diego, where an upper limit is given higher than any we used. On Figure 5 is presented a series of points determined similarly to those of Figures 3 and 4, for El Paso, Tex., using Figures 8 and 9 of Supplement 16, which are presented for use within certain dew-point limits; considering dew points 15, 25, and 40, we get points that lie on or close to a straight line at higher temperatures, but note considerable divergence at low.

It must be borne in mind that the points we have plotted on Figures 3, 4, and 5 will give, under given conditions of dew point and relative humidity, exactly the same indicated minimum temperatures as will the hygrometric charts from which these points have been



FIGURES 3 and 4.—Relation between current temperature and ensuing minimum temperature, derived from hygrometric curves by graphical method; dew points constant

derived. We have shown results of keeping the dew point constant [3] and allowing the relative humidity (and, consequently, the current temperature) to vary; let us now keep the temperature constant and show results of allowing the dew point (and therefore the relative humidity) to vary.

[3] Because of criticisms that have been made, it seems necessary to state that we have not stated nor assumed constancy of dew point through the night. The terms "constant" and "vary" refer, of course, to employing the same or different values in our formulas.

EFFECT OF CHANGING DEW POINT ONLY

For instance, take the chart for Grand Junction, already used. The effect of keeping the dew point constant at either 30° or 40° is shown on Figure 3. If we keep the current temperature constant at 53 and allow the dew point to vary, the relative humidity, at 5° dew-point intervals, is given by the abscissas of the points of intersection of the line A with the dew-point curves on Figure 2. Then, using the dew points and relative humidity values of these points of intersection, we find the indicated ensuing minimum temperature from the Grand Junction hygrometric curve.

These minima are plotted on Figure 2, each below its respective point on line A with which it is connected by a dotted line to indicate the total nocturnal fall of temperature. Humidities above 55 per cent were not used, as per Grand Junction chart. The minima lie on the line A_g, which is nearly straight and horizontal, indicating that (under the conditions for which the Grand Junction chart was prepared) a given noon temperature is followed by practically the same minimum the next morning, even though the noon dew point may have any value from 15° to 35°. Line B and line C, drawn at 70° and 80°, respectively, furnish bases for lines B_g and C_g, on Figure 2, obtained similar to line A_g by use of the Grand Junction chart. B_g and C_g show that, while the ensuing minimum varies more than in the first case, the variations are relatively slight. Thus, with a noon temperature of 70°, the indicated minimum changes only 4° when the dew point at noon has values from 15° to 50°.

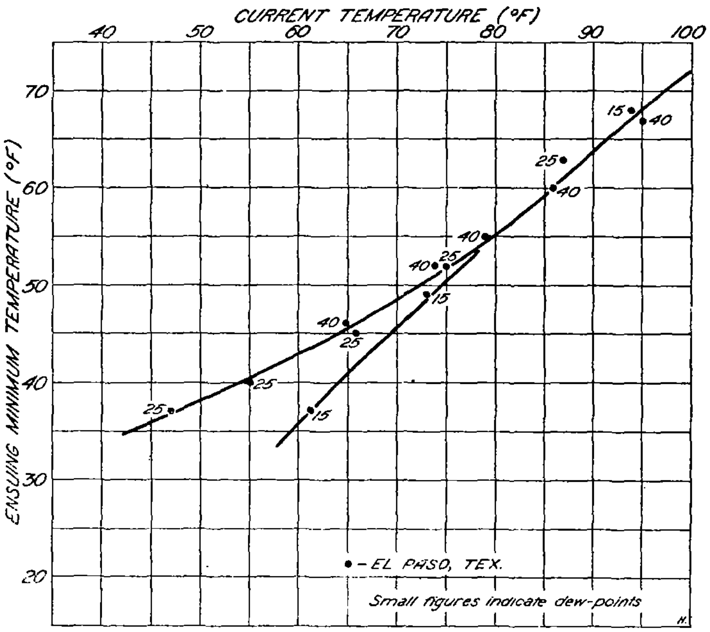


FIGURE 5.—Relation between current temperature and ensuing minimum temperature derived by graphical method from hygrometric curves for El Paso, Tex.; at certain dew points

COMPARISON OF DEW-POINT AND TEMPERATURE EFFECTS

Using the same San Diego chart as before, we find that a current temperature of 61° and dew point 30° indicate a minimum of 47°, which is plotted on Figure 6 (S. D.). Then, keeping the dew-point constant at 30, we raise the current temperature 5° to 66°, for which the indicated minimum is 50.5, which is plotted on the fifth ordinate to the right of the one first used. Then lowering the current temperature to 56, we find the indicated minimum to be 41, which is plotted on the fifth ordinate to the left of the original. Then the line through these 3 points shows the rate of change of minimum when current evening temperature varies with constant dew evening point, 30°. Similarly, we find the indicated minimum when the temperature is kept constant at 61 while the dew point is raised 5 above and then lowered 5 below 30°, and plot these minima on the fifth ordinates to the right and left of the original, as before. Then the line drawn, dotted, through these two points and the starting point (47 on the original ordinate) shows the rate of change of minimum when the dew point has different values, current temperature remaining constant. By comparing the slopes of the two lines we have now obtained we may compare dew point effect with that of current temperature. Similar lines, using the same dew points and current temperatures, have been drawn on Figure 6 for Grand Junction, Colo., San Jose, Calif., El Paso, Tex., Phoenix, Ariz., and Medford, Oreg., together with other similar diagrams using other basic dew points and temperatures, including two diagrams for Boise, Idaho.

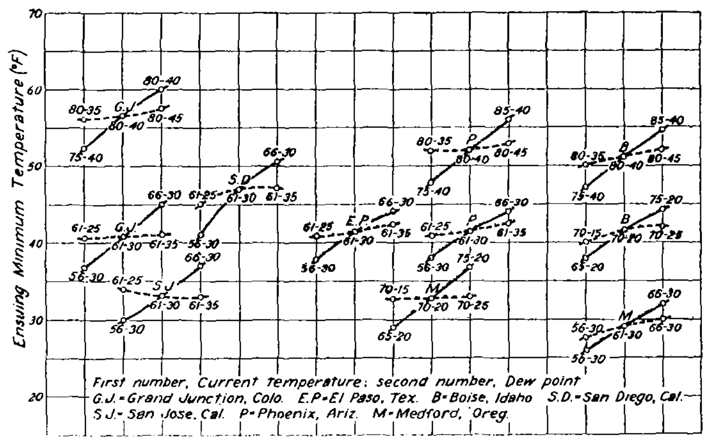


FIGURE 6.—Comparison of dew-point and temperature effects on ensuing minimum temperature; derived from hygrometric curves by graphical method, at 5° intervals

The temperature and the dew-point effects here shown are, of course, the total effects due to changes in these elements. Thus, we have not attempted to distinguish between the nocturnal cooling due to local radiation and that due to importation of cold air; the formula does not make this distinction. Neither is there a distinction made between the effect of atmospheric moisture, as indicated by the dew point, in retarding nocturnal cooling by checking radiation on the one hand from the effects of such moisture in liberating latent heat during condensation or freezing. These special effects may, however, be studied by classification and preparing special charts or curves (as described below), separating, e. g., "radiation nights" from "cold-wave nights" and nights on which dew, fog, or frost forms from dry nights.

In each diagram of Figure 6 the slope of the temperature line is much greater than that of the dew-point line. The former slope is 0.6 or more, indicating that a change of 10°, for instance, in the current temperature is followed by a change of at least 6° (in the same direction) in the ensuing minimum. The dew-point line slope is generally not above 0.1 or 0.2, sometimes being 0 or even negative, indicating that a change of 10° in the dew-point is generally accompanied by a change of not more than 1° or 2°, sometimes no change at all, in the ensuing minimum. We conclude, therefore, that current temperature is generally much more important than the dew-point in hygrometric formulas. In other words, at the stations, during the times of year and under the meteorological conditions for which the hygrometric charts we have transformed and otherwise examined were prepared, the minimum temperature is higher on one day than on another mainly because the preceding day or evening was warmer in the former case than in the latter, rather than because of differing evening dew points.

SYSTEM BASED ON PRECEDING TEMPERATURES

Since the minimum results from cooling from some higher preceding temperature, and since we have shown that changes in the latter greatly affect the ensuing minimum, we may logically advocate the plotting directly of the relation between current and ensuing minimum temperatures, thus obtaining such curves as those of Figures 3, 4, and 5. Since we obtain the minimum directly, and since straight-line relations are indicated, the method will be simpler than the hygrometric.

We should expect that, since our curves of Figures 3, 4, and 5 have been derived by an indirect method, curves that would fit the original data still better could be obtained directly. The temperatures used as the independent variable on these charts are the current temperatures at the times of observations, which differ considerably among the several stations considered, according to local mean time and with reference to the times of sunset, etc. At Grand Junction we used observations taken at local mean noon; at Medford, at 5 p. m., Pacific standard time; at San Jose, at 4:32 p. m., local mean time; at other stations, evidently at the time of the p. m. simultaneous observations of the Weather Bureau, which are later, according to local time, than are those at San Jose.

Although the time of maximum temperature varies more or less from day to day, even in fair weather, we may consider the prediction of the minimum from the preceding maximum temperature as based on the current temperature at the time of maximum, and thus include the maximum-minimum under the general method based on preceding temperatures. Since the maximum-minimum was proposed by me many years ago (in 1911, in the same paper in which my experiences with the dew-point formula were reported), and has since then been successfully used by me and others in actual forecasting, we may properly consider that system first. It has been explained since 1911, with some modifications and additions (**2**), (**3**), and (**6**).

The relation between maximum and ensuing minimum temperature was plotted on dot charts, a separate chart for several types of weather determined according to cloudiness, pressure, and wind conditions. Since straight-line relations were found (as we have herein determined indirectly for current temperatures generally) the several formulas are of the simplest type, $Y=bX+a$, where X is the daily maximum temperature, a and b are constants, and Y is the ensuing minimum temperature. A series of corrections, one for each of four groups of dew-point values, provided for variations in atmospheric cooling. The fact that later investigations at Grand Junction showed that the dew-point effect is unimportant in the hygrometric formulas for that station agrees with results we have obtained in transforming one of those formulas. (See (**6**), p. 41.) Thus the maximum-minimum method as proposed by me provided for the effects of "other conditions" besides the maximum temperature upon the following minimum, dew point being specifically included.

The maximum-minimum formulas and charts derived for Grand Junction and substations in vicinity thereof were successfully used in minimum-temperature forecasting (in connection with hygrometric formulas) for several years by me; also, they were so employed by my successor, Mr. A. M. Hamrick (**6**) and (**7**). I have successfully used a similar method at San Jose, Calif., during the past eight years. And Cook found the maximum-minimum relation valuable in the Red River Valley of the North (**8**), although he did not, apparently, consider dew-point effects or classify according to weather types.

ELLISON'S CRITICISM OF THE MAXIMUM-MINIMUM METHOD UNWARRANTED

Ellison has, therefore, adopted another untenable position when he states that the maximum-minimum formulas "appear to be faulty at their source" because "with any given maximum temperature a variety of values of absolute humidity are observed in practice * * * and consequently a variety of minimum temperatures are experienced." (**5**), p. 488. A system based on faulty principles could not be successfully used in practical work, as has the one in question. Also, the dew-point corrections, which provide for any possible absolute humidity effects, were included in the system as described in my papers (**3**) and (**6**) referred to by him under Nos. 22 and 31. Further, it is believed that my "application of a complex system of type classification" "in the endeavor to improve the usefulness of the basic relationship" of the maximum-minimum method (as well as of the hygrometric) should be still further extended; since neither hygrometric nor current-temperature system shows directly the effects of special values of other than basic conditions; whereas these effects can be readily shown by special charts, curves, or corrections for such special values, as is illustrated below.

THE GENERAL METHOD BASED ON PREVIOUS TEMPERATURES

In deriving other types of current-temperature formulas from original data, as suggested above, our methods may be similar to those used with the maximum-minimum. That is, we plot on a dot chart the individual observations, which may have been previously classified according to some system found desirable, current temperatures preferably as abscissas and ensuing minimum temperatures as ordinates. In order to determine a desirable classification system the dots for each observation may be entered in such a manner, and may be accompanied by such auxiliary entries, as to indicate the accompanying conditions that might affect the minimum temperature.

CLASSIFICATION ACCORDING TO TYPES OF CONDITIONS

For example, on Figure 7 entries have been made, based on the regular post meridian observations at San Jose, Calif., during April for five years. Cases when skies were clear are indicated by being plotted with small circles, while cloudy cases are entered with dots. In each case the post meridian dew point is entered at its proper cross or dot on the chart. Plainly we see that the clear cases tend to be grouped about a straight line, while the cloudy cases are relatively scattered and mostly at higher levels. It is also noted that when the dew point is below 40° the minimum temperature is usually lower than in other cases.

As the next step we may make separate charts for cloudy and for clear cases, using data for other Aprils in order to have a good supply of dots; we may well prepare two clear charts, one for cases with dew point above, and one for cases with dew point below, 40°. We may have a separate chart for cases in which the sky was clear in the evening but became cloudy during night, and another for the reverse, etc. And at each point on these charts we may enter, or indicate by symbols or by colored ink or in other ways, the dew point, the barometric pressure, the wind, the soil conditions, precipitation, fog, or any other data that may possibly show effect on the minimum temperature; not only evening values, but also changes, of certain conditions may be valuable.

Possibly simply a cursory examination will determine the influence of some factor, as in the case of clear weather on our chart. It may be necessary to compute averages for certain elements; for instance, we might compute the average dew point for cases plotted in each 5° square, and thus determine the dew-point effect. (See also above, under Preferred Method.) Such possible factors as our trial charts show to be ineffective may now be discarded, and we may prepare a final set for charts, a separate chart for each weather type formed by combining the

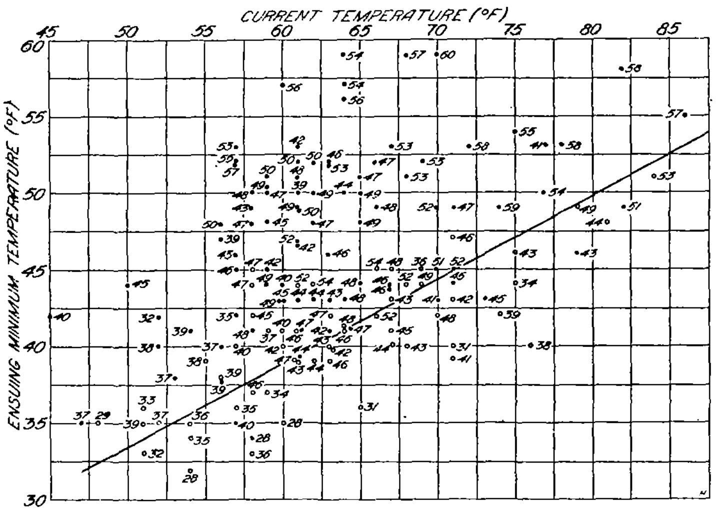


FIGURE 7.—Relation between current evening temperature and ensuing minimum temperature at San Jose, Calif., during April

elements we have found effective, or a combined chart for such types as can readily be combined.

We then draw a line of best fit to each group of plotted data. If types be combined we may have two or more lines fitting different portions of the chart. For instance, if we combine on one chart cases for all dew points that occur, we may have a line for the lower dew points and one or more other for higher dew points.

Even after progressing thus far we find that the plotted dots do not all lie on the line of best fit, though they are on an average closer than were those on the original chart to its best line; we still find that a given current temperature was followed in different cases by different minima. By study of the individual cases represented by the individual dots, especially those farthest removed from the line of best fit, we may discover the causes of the observed variations; then we may form a correction law or curve or, if data be sufficient we may draw a special chart or charts showing the effects of the newly discovered cause. Thus the average variation of dots from the line of best fit will be decreased as our classification becomes more complex.

CONCLUSION

We have considered at the beginning of this paper the dew-point formula or relation, in which the dew point is considered all important. Then we examined the hygrometric formulas and curves, in which the dew point is nominally of great importance but actually, as we have found, of secondary effect. Finally we have the temperature formulas, in which dew point is assigned definitely a secondary place, although provision is made for any moisture effects that may occur. We may have been surprised to find that the dew point does not appear prominently on our charts, in view of the undoubted importance of water vapor in retarding radiation and of the large amount of latent heat liberated during condensation and freezing. Experience has shown that minimum-temperature formulas do not give sufficiently accurate indications for satisfactory work in actual forecasting, unless modified; and minimum-temperature forecasters generally consider formulas as only approximate, modifying results according to special rules or "experience." Classification of conditions and the use of auxiliary curves furnish a means of systematic mathematical expression, quantitatively, of any modifying influence that may be discovered at any station in any formula, whether by study of weather maps or otherwise. Thus any forecaster may make his discoveries available to all students of minimum temperature forecasting, and the necessity of "esoteric interpretation" of modifying influences may be reduced. (5, p. 486.)

The general method of predicting minimum temperature from preceding temperature is logical and direct. The effect of any other independent variable upon the minimum temperature may be shown by classification of cases into groups; and this classification may be enlarged or narrowed as conditions at any particular station may require. Thus the method has characteristics fitting it for general use. Even though complicated classification may be required in some cases, the system is as simple as physical conditions permit, since if any effective variable condition is not provided for in a forecasting system, that system will not be as accurate as possible in actual use. The effects of conditions found important may be shown on separate charts or by auxiliary or correction curves, which will or will not be parallel to the basic curve according to indications of data used.

If we find in any case a straight-line relation, the corresponding formula is simple and its use is convenient. If our line be parabolic or hyperbolic, or of some other fairly simple form, use of the formula is somewhat inconvenient. If we have, also, a set of numerical corrections, inconvenience is considerably increased thereby. If we use instead of the formula, the dot chart itself, on which has been drawn, perhaps free-hand, the best fitting curve, whose equation need not be known, we not only obtain immediately our result, but also observe what dependence may be placed on different parts of the curve. If in addition we enter additional information relative to special conditions that preceded certain minima (especially for points farthest from the curve) we may be able to improve our forecasts still further. Our suggested methods imply a large amount of preliminary labor in studying weather maps and data; but the forecaster needs all possible assistance in the complicated and difficult problem of minimum-temperature prediction.

As additional information in connection with this paper I include herewith a chart, Figure 8, showing the relation between the daily maximum temperature at the Weather Bureau Office, Mobile, Ala., and the ensuing morning minimum temperature at one of the fruit-frost stations at Seven Hills, Ala., 18 miles distant; using data for the winter of 1929–30 (Nov. 19–Feb. 18, 3 months). This is a chart using all data in accordance with the suggested system; and shows that the maximum-minimum form of the general temperature method is applicable not only in the Far West, but in the humid Gulf Coast region.

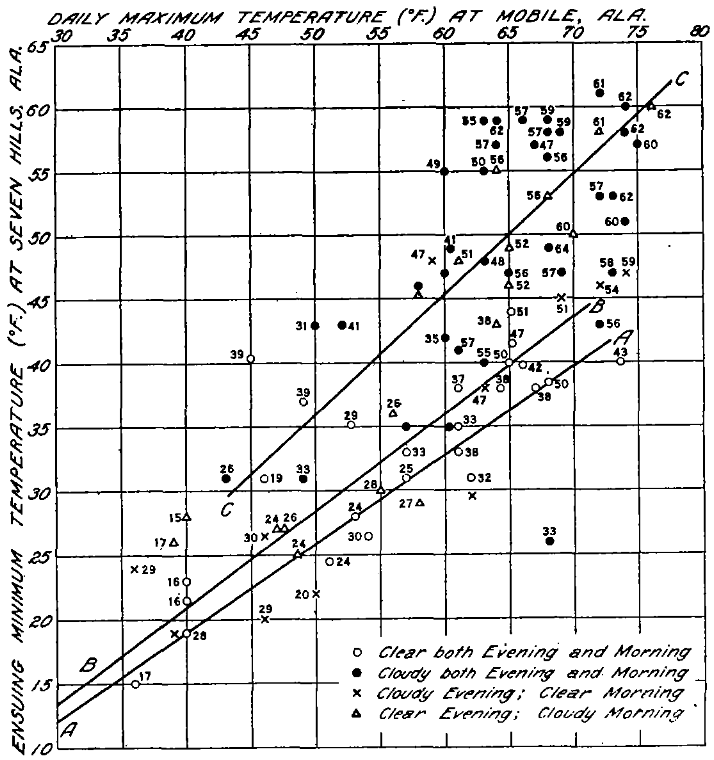


FIGURE 8.—Relation between the daily maximum temperature at the Weather Bureau Office, Mobile, Ala., and the ensuing minimum temperature at Seven Hills, Ala., Nov. 19, 1929, to Feb. 18, 1930

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DISCUSSION

By FLOYD D. YOUNG

[Weather Bureau Office, Medford, Oreg., April 30, 1930]

A great deal has been written about formulas of different types for use in minimum temperature forecasting, but very little has been said about the difficulties in the way of applying formulas in actual practice. The impression has become rather general that this type of forecasting is a simple matter, presenting few, if any, of the difficulties met with in the preparation of other types of weather forecasts. The feeling has been that weather conditions in the fruit-growing valleys of the Pacific Coast States are "settled" and that satisfactory minimum temperature forecasting is only a matter of applying a formula based on an observation made at some time in the afternoon or evening.

The initial experience of the writer with the hygrometric formula at Medford in 1917 (1) inclined him somewhat toward this belief, but it was soon recognized that the formula was valuable only as a starting point in practical forecasting. The experience of 14 years in this work, covering 16 different fruit-frost districts, has served to strengthen this conclusion. Most minimum temperature formulas are based altogether on data for nights when the temperature actually falls to the freezing point or lower in the district for which it is developed. Such nights usually make up only a very small percentage of the total during a frost season. On most nights the temperature is prevented from falling to the freezing point by wind, clouds, fog, or other agency.

Even if we could develp a formula or series of formulas that would give perfect results on frosty nights, we still would be faced with the necessity for determining whether the formula would have application on a particular night. Herein lies one of the most difficult problems in minimum temperature forecasting. The practical prediction of minimum temperatures presents different problems in different districts. Forecasting is more difficult in some districts than in others, but every section has its own peculiar local conditions which must be thoroughly understood before the forecaster can feel that he is approaching the maximum possible accuracy.

If all the nights during the frost season were calm and clear, temperature ranges between maximum and minimum could be determined with mathematical precision. However, it is seldom indeed that we approach these ideal conditions. For example, at Medford there are few frosty nights during the spring frost season without some cloudiness. It may be cloudy at the time the forecast is made, or even raining or snowing, with the current temperature in the lower thirties. If the sky clears, as it often does, it is sure to freeze; if it remains overcast there will be no danger. If a study of the weather map and all other available data brings the decision that the sky will clear before morning, the question arises "When will the sky clear?" If it clears at 9 p. m., the temperature obviously will fall lower than it will if clearing comes at 2 or 3 a. m.

In southern California the opposite condition is often met. The sky is clear and the current temperature is low at the time the forecast is made, but the barometer is falling and the weather map shows a low-pressure area centered over northern California and spreading rapidly southward. The sky is almost sure to become overcast before sunrise, but when will the clouds form? They have come too late on many nights to prevent a fall in temperature below the danger point. On some such nights weather conditions have changed so rapidly that it has been necessary to extinguish the orchard heaters in a downpour of rain. When the temperature falls steadily at the rate of 3° or 4° an hour as long as the sky remains clear, an error of several hours in estimating the time of clouding over will give the forecaster's reputation a severe setback.

On another evening in southern California a desert wind may be blowing. Its velocity is anywhere from 15 to 30 miles per hour. The dew point is perhaps 10° above zero and the current temperature is in the forties. As long as the wind continues the temperature will remain practically stationary. If the wind lulls, the temperature will fall with almost unbelievable rapidity. The cessation of these winds is often sudden and complete. If the forecaster decides the wind will not last through the night, he must make some sort of an estimate of the time it will cease.

In the San Joaquin Valley, Calif., dense fog may form over a part of the valley floor during the night, but the hillsides and other parts of the valley floor remain clear. The problem, then, is to decide where and what time of night the fog will form, and how far under the fog-covered area the influence of the cold air draining from the slopes will extend.

These are a few of the difficulties which must be faced in making practical minimum temperature forecasts. There are many others of lesser importance. The solution of these problems will be found only in an intelligent use of the weather map and aerological data, together with as complete a knowledge of the district as possible. Formulas are of little value in dealing with these special conditions. The writer does not agree with Mr. Nichols' statement that "classification of conditions and the use of auxiliary curves furnish a means of systematic mathematical expression, quantitatively, of any modifying influence that may be discovered at any station in any formula." The statement may be true in a literal sense, but the number of classifications and auxiliary curves necessary would approximate the number of nights on which the special conditions prevailed. In the writer's opinion it is utterly impracticable to reduce all minimum temperature forecasting factors to a mathematical basis. A dozen or more curves may be drawn to represent different conditions or different influences which may affect the minimum temperature, but it will still be necessary for the forecaster to decide which one of these curves he shall use on a particular night. It is almost as important to issue a reassuring forecast when local conditions alone are threatening, as it is to issue a warning when crop protection is necessary.

The writer's experience has been that the most successful and practical method of making minimum temperature forecasts is to develop a formula based on all available data to indicate what the minimum temperature will be under more or less ideal conditions, that is, when the temperature fall after sunset is least affected by clouds, wind, or other such factors. The effect of modifying influences is then estimated from the weather map and all other sources at hand, and the formula estimated is raised or lowered accordingly.

In the paper under discussion the author has demonstrated mathematically the relation between the hygrometric formula and the current temperature formula and intimates that the importance of the dew point has been exaggerated in the hygrometric formula. However, it is apparent without mathematical demonstration that the association of the dew point and relative humidity gives a value for the current temperature at the time of observation, and this point was brought out in the writer's paper, Forecasting Minimum Temperatures in Oregon and California (1) published in 1919. The relation between the temperature of a radiating surface and the rate of radiation loss of heat to a clear sky is so fundamental that it is difficult to see how anyone could fail to recognize that current temperature at some time during the afternoon or evening is the important factor in determining the amount of fall in temperature on a clear, calm night. Under the caption Factors Which Determine Minimum Temperature Represented in Hygrometric Formula the more important factors which influence the rate of effective radiation from the earth during the night are given in the order or their importance in the paper before cited. (1) On many nights, especially in southern California, the temperature of the dew point is far below any possible minimum temperature. In such cases there will be no liberation of latent heat of vaporization at all, and the dew point value is important in itself only as an index to the rate of loss of heat by radiation to the clear sky. An unusually low evening dew point in southern California often is the best possible evidence that the wind will continue to blow throughout the night, with no danger of frost. A rising dew point in the early evening usually presages a lessening wind velocity and freezing temperatures.

On the other hand, higher dew points are usually important in determining the temperature fall. For example, the temperature has never fallen to 32° F. or lower in the Pomona fruit-frost district when the 4:40 p. m. dew point has been 50° F. or above during the past 14 winters, no matter what the current weather conditions have been at the time of observation. During the same period there has never been a freezing temperature in the Medford district during the spring frost season when the 4:40 p. m. dew point has been 45° F. or higher.

In connection with the statement in the conclusion of the paper under discussion "We may have been surprised to find that the dew-point effect does not appear more prominently on our charts, in view of the undoubted importance of water vapor in retarding radiation and of the large amount of latent heat liberated during condensation and freezing," attention might be called to studies of changes in dew point during the night at Medford and Pomona discussed briefly in the writer's paper in Supplement 16 of the MONTHLY WEATHER REVIEW. (1) It was shown that rapid changes in dew point at Medford after the evening observation are the rule under certain conditions, and that it is unsafe to draw any conclusions regarding the relative effects of high or low dew points on the nocturnal temperature fall from 4:40 p. m. data.

For example, the dew point on April 27, 1918, was 11° F. at 4 p. m. and 22° F at 5 p. m. By 7:30 p. m. it had risen to 35° F. During the rest of the night it ranged between 25° and 30° F. On the night of April 25, 1918, the dew point was 30° F. at 4 p. m. and 26° F. at 5 p. m., and varied between 26° and 32° F. during the night. In these two cases the effect of atmospheric moisture in retarding radiation, and the amount of heat liberated in condensation and freezing were practically the same, yet the values of the afternoon dew-point temperatures would indicate widely different effects.

It is believed that an analysis of the so-called Young hygrometric formula, described in (**1**), modifications of which are being used in fifteen different fruit-frost districts on the Pacific coast, will disprove the statement that the dew point is "nominally of great importance," which appears in the conclusion of the paper under discussion. Moisture effects are given their proper weight in the formula and no more.

The paper under discussion is an interesting and valuable theoretical discussion of minimum temperature-forecasting methods, in which points which have been accepted, as self-evident or not explained in detail heretofore, are brought out by mathematical anlaysis. However, in the writer's opinion, based on experience in minimum temperature forecasting in the Pacific Coast States, the forecasting methods suggested by the author are considerably more complicated than those now in use, and will not increase the accuracy of the results being obtained with present methods. It is the writer's contention that while formulas developed from data secured on "radiation" nights, or nights on which the temperature fell to or near the freezing point are of great value in minimum temperature forecasting, it is just as impracticable with our present knowledge of forecasting to use formulas to determine when wind or clouds will prevent a fall in temperature to the freezing point or when an influx of cold, dry air will cause the temperature to fall unusually low, as it is to forecast rains or gales at any given point by formula. Successful noon forecasts of minimum temperature for the following morning at Pomona have been made for several years when general orchard heating is in prospect, through the use of the morning weather chart alone, without the use of a formula.

In the preparation of minimum temperature forecasts in the evening, the time factor is important, since the later the forecasts are given to the public the less valuable they become. The system of forecasting used should be as simple as possible without sacrificing accuracy. The type of formula used, whether curves or equations with corrections, is entirely a matter of the personal preference of the forecaster, and is unimportant from the standpoint of results obtained.

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(**1**) Forecasting Minimum Temperatures in Oregon and California, by Floyd D. Young. MONTHLY WEATHER REVIEW, Supplement No. 16, pp. 53–55.

REJOINDER

By ESEK S. NICHOLS, San Jose, Calif., May 23, 1930

Referring to discussion by Mr. Floyd D. Young:

Mr. Young has well emphasized the complications and difficulties of minimum-temperature forecasting, which have occasioned the large amount of study that has been devoted to the subject without, we may say, completely solving our problem. Also, he has very properly emphasized the use of the weather map, which is of course indispensable.

In his ninth paragraph he advises raising or lowering the formula estimate according to estimated effects of modifying influences. This implies a quantitative estimate in degrees of such effects, and it is difficult to see why an auxilliary curve or curves can not be drawn to express them. Consider, for example, the clearing conditions at Medford referred to in the last sentence of his fourth paragraph. We should have an auxilliary curve for cases when the sky clears at about 9 p. m. and another curve for use if clearing occur at 2 or 3 a. m.; deciding in the evening which if either curve to use, after considering the weather map and other available helps.

Evidently he overestimates the number of classifications and auxilliary curves that would be required; for modifying conditions fall into great classes or types, as do conditions producing rainfall, for example. Classification may be based on weather map wholly or in part; see, for example, types for Grand Junction, Colo. (**3**) and (**6**).

Mr. Young's statement regarding the importance of current temperature in determining nocturnal cooling is exactly in accord with my remark that the dew point is "nominally" of great importance in hygrometric formulas; since dew point appears prominently whereas current temperature does not specifically appear in those formulas. Also, his statement accords with my paper as a whole, since my principal purpose is, as indicated in the title and in the conclusion, to advocate the predicting of minimum temperatures from preceding, or current, temperatures and to develop a method of predicting on that basis. This paper is presented as outlining a general method of attacking the problem of minimum temperature forecasting.